

Curriculum MAT 232 (Spring 2017)
Functional Analysis
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Lecture Structure

1 Banach spaces (Chapter 1 in [12])

- Normed spaces (Sect. 1.1).
- Completeness, Banach spaces (Sect. 1.2).
- Continuous functions, contractions (Sect. 1.3), Banach fix point theorem. Applications (Sect. 4.2).
- Compactness and finite dimensional spaces (Sect. 1.4).
- Linear and continuous functions (Sect. 1.5). Applications (Sect. 4.3).
- Zorn's lemma, Hamel bases and the Hahn-Banach theorem (Sect. 1.6). Quotient spaces.
- The interior mapping and closed mapping theorems (Sect. 1.7).
- Baire theorem and uniform boundedness (Sect. 1.8)
- Weak convergence (Sect. 1.9).
- Reflexive spaces (Sect. 1.10).

2 Hilbert spaces (Chapter 2 in [12])

- Geometry (Sect. 2.1).
- Orthogonality and Bases (Sect. 2.2).
- Linear and continuous functions (Sect. 2.3).
- Spectral theory (Sect. 2.4).

3 Numerical methods (Sections 4.1-4.5 in [12])

- Discretization (Sect. 4.1).
- Projections and projection methods (Sect. 4.4).
- Galerkin method (Sect. 4.5).

4 Additional topics (Sections 5.1, 6.8, 7.1 and 7.4 in [12])

- Distributions.
- Sobolev spaces.
- Fixed point theorems.
- Arzela-Ascoli theorems.

References

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